Math 117: Mt 2 Reflection

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Definition of convergent sequence

Correct definition:

A sequence (S_n) is said to *converge* to the real number L if for any $\epsilon>0$ there exists a number N such that $n>N$ implies $|S_n-L|<\epsilon$.

Incorrect(my) definition:

A sequence (S_n) is said to *converge* if for any $\epsilon>0$, there exists a real number L such that there exist a number N, $n>N$ implies $|S_n-L|<\epsilon$.

Why is it wrong

Incorrect(my) definition:

A sequence (S_n) is said to *converge* if for any ϵ>0, there exists a real number L such that there exist a number N, n>N implies |S_n-L|<ϵ.

❌***L* depends on ϵ**

✅**Should be:**

- **ϵ is free**
- ***L* is free**
- **N depends on ϵ**

If (s_n) converges to s and (t_n) converges to t, use $(\epsilon\text{-}N)$ language to prove that $(s_n \cdot t_n)$ converges to st.

Let $\varepsilon > 0$. Since all convergent sequences are bounded, there is a constant $M > 0$ such that $|s_n| \leq M$ for all n.

Since $\lim t_n = t$, there exists N_1 such that $n > N_1$ implies $|t_n - t| < \varepsilon/2M$.

Also, since $\lim s_n = s$ there exists N_2 such that $n > N_2$ implies

$|s_n - s| < \varepsilon/2(|t| + 1)$

(Note: We use $\varepsilon/(2(|t| + 1))$ instead of $\varepsilon/(2|t|)$ in case $t=0$)

Now if $N = max\{N_1, N_2\}$, then $n > N$ implies

$|s_n \cdot t_n - st|$

$\leq |s_n| \cdot |t_n - t| + |t| \cdot |s_n - s|$

$\leq M \cdot \varepsilon/2M + |t| \cdot \varepsilon/2(|t| + 1)$

$< \varepsilon/2 + \varepsilon/2$

$= \varepsilon.$

What I misunderstood

- M is a constant, and ***M*>0**
- We could add one (or any possible positive value) to the denominator to make sure it's not zero.